Further Discussion of Two-Phase Flow in Porous Media

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In two previous papers (Slattery, 1968 and 1970), the concept of local volume averaging was used to develop from first principles the equations appropriate for describing the flow of two or more phases through a porous medium. In what follows, we would like to correct an error in that discussion. As the result of this correction, we recommend an approximate form of the local volume averaged equation of motion that is consistent with Muskat's extension of Darcy's law to multiphase flow in porous media (Muskat et al., 1937; Scheidegger, 1960).

After inertial effects and a portion of the viscous terms were neglected, the local volume averaged equation of motion for phase *i* was found to take the form (Slattery, 1970; see Equation (24)*)

$$\nabla \overline{(\mathcal{P} - p_0)}^{\,i} + \mathbf{g}_i = 0 \tag{1}$$

where

$$\begin{aligned} \overline{(\mathcal{P} - p_0)}^{\,i} &\equiv \frac{1}{V} \int_{V_i} (\mathcal{P} - p_0)\; dV \\ &= \psi s_i <\mathcal{P} - p_0>^i \\ &= \psi s_i \,(<\mathcal{P}>^i - p_0) \end{aligned} \tag{2}$$

and

$$<\mathcal{P}>^i \equiv \frac{1}{V_i} \int_{V_i} \mathcal{P}\; dV \tag{3}$$

$$\mathbf{g}_i \equiv -\frac{1}{V} \int_{S_{mi}+S_{fi}} [\mathbf{T} + (p_0 - \rho_i \varphi)\mathbf{I}] \cdot \mathbf{n}\; dS \tag{4}$$

Physically, $\mathbf{g}_i$ is the local force per unit volume that phase *i* exerts upon both the pore walls and the other fluid phases present beyond the hydrostatic force and any force attributable to a constant reference or ambient pressure p_0. After substituting Equation (2) into Equation (1), we find

$$\psi s_i \,\nabla <\mathcal{P}>^i + (<\mathcal{P}>^i - p_0)\, \nabla(\psi s_i) + \mathbf{g}_i = 0 \tag{5}$$

Previously, $\mathbf{g}_i$ was taken to be independent of p_0 [Slattery, 1970; see Equation (15)]. This would imply from Equation (5) that $<\mathcal{P}>^i$, $\overline{\mathbf{v}}^{\,i}$ and s_i are dependent upon p_0. This can not be correct. The constant reference pressure p_0 was introduced in the analysis for convenience; it may be assigned any arbitrary value including zero.

In order to correct this error, let us express

$$\mathbf{g}_i = \mathbf{h}_i + \frac{1}{V} \int_{S_{mi}+S_{fi}} (\mathcal{P}^i - p_0)\mathbf{n}\; dS \tag{6}$$

where $\mathbf{h}_i$ is the viscous force per unit volume that phase *i* exerts upon the pore walls and the other phases present:

$$\mathbf{h}_i \equiv -\frac{1}{V} \int_{S_{mi}+S_{fi}} (\mathbf{T} + p\mathbf{I}) \cdot \mathbf{n}\; dS \tag{7}$$

We can also observe that, for any constant *c*, a previously developed theorem for the local volume average of a gradient requires (Slattery, 1967 and 1972)

$$\begin{aligned} \overline{\nabla c}^{\,i} &= \nabla \overline{c}^{\,i} + \frac{1}{V} \int_{S_{mi}+S_{fi}} c\; \mathbf{n}\; dS \\ &= c \nabla(\psi s_i) + \frac{c}{V} \int_{S_{mi}+S_{fi}} \mathbf{n}\; dS \\ &= 0 \end{aligned} \tag{8}$$

or

$$\nabla(\psi s_i) = -\frac{1}{V} \int_{S_{mi}+S_{fi}} \mathbf{n}\; dS \tag{9}$$

Substituting Equations (6) and (9) into Equation (5), we see

$$\psi s_i \,\nabla <\mathcal{P}>^i + \frac{1}{V} \int_{S_{mi}+S_{fi}} (\mathcal{P}^i - <\mathcal{P}>^i)\, \mathbf{n}\; dS + \mathbf{h}_i = 0 \tag{10}$$

It seems reasonable to neglect the second term on the left of Equation (10) and to write instead

$$\psi s_i \,\nabla <\mathcal{P}>^i + \mathbf{h}_i = 0 \tag{11}$$

We can now develop empiricisms for $\mathbf{h}_i$ in the same way that we previously proposed empirical relations for $\mathbf{g}_i$ in Equation (1) (Slattery, 1970). For example, in a stationary nonoriented (Slattery, 1972) or isotropic porous structure it may be reasonable to say

$$\mathbf{h}_i = N_i \overline{\mathbf{v}}^{\,i} \tag{12}$$

where N_i is normally taken as a function of s_i. When $\mathbf{h}_i$ is represented by Equation (12), Equation (11) takes the form of Muskat's extension of Darcy's law to multiphase flow in porous media (Muskat et al., 1937; Scheidegger, 1960):

$$\psi s_i \,\nabla <\mathcal{P}>^i + N_i \overline{\mathbf{v}}^{\,i} = 0 \tag{13}$$

In agreement with previously reported successful applications of Equation (13), (Rapoport and Leas, 1953; Blair and Weinaug, 1969; Douglas et al., 1959), we have found it capable of representing very accurately some data reported by Terwilliger et al. (1951).

NOTATION

$\mathbf{g}_i$ = force per unit volume that phase *i* exerts upon the pore walls and the other fluid phases contained within S beyond the hydrostatic force and beyond any force attributable to the ambient pressure

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* In the paper by Slattery (1970), Equation (22) and the sentence following Equation (22) should be revised in this way:

$$\overline{(\mathbf{T} + p\mathbf{I})}^{\,i} = \mu_i \left[\nabla \overline{\mathbf{v}}^{\,i} + (\nabla \overline{\mathbf{v}}^{\,i})^t \right] + \frac{\mu_i}{V} \int_{S_{mi}} (\mathbf{v}^i\, \mathbf{n} + \mathbf{n}\, \mathbf{v}^i) dS \tag{22}$$

Let us define

$$\mathbf{j}_i \equiv -\mu_i \operatorname{div} \left(\frac{1}{V} \int_{S_{mi}} [\mathbf{v}^i \mathbf{n} + \mathbf{n} \mathbf{v}^i]\; dS \right) \tag{22a}$$

If we assume by analogy with Equation (18) that $\mathbf{g}_i + \mathbf{j}_i$ may be represented by an empiricism of the form

$$\mathbf{g}_i + \mathbf{j}_i = M_i \overline{\mathbf{v}}^{\,i} \tag{22b}$$

Equation (8) becomes for such a fluid

$$\nabla \overline{(\mathcal{P} - p_0)}^{\,i} - \mu_i [\operatorname{div}(\nabla \overline{\mathbf{v}}^{\,i}) + \nabla(\operatorname{div} \overline{\mathbf{v}}^{\,i})] + M_i \overline{\mathbf{v}}^{\,i} = 0 \tag{23}$$

$\mathbf{h}_i$ = viscous force per unit volume that phase i exerts upon the pore walls and the other fluid phases contained within S. Defined by Equation (7)
$\mathbf{I}$ = identity tensor
$\mathbf{n}$ = unit normal that is outwardly directed with respect to S_i
p = pressure
p_0 = constant reference pressure
$\mathcal{P}$ = modified pressure defined by $\mathcal{P} \equiv p + \rho\varphi$
s_i = local saturation of phase i. If there are n fluid phases present then $s_i \equiv \dfrac{V_i}{\sum_{j=1}^{n} V_j}$
S = an arbitrary closed surface that is associated with every point in the porous medium
S_i = closed boundary surface of V_i
S_{fi} = portion of S_i that coincides with the pore walls
S_{mi} = portion of S_i that coincides with the boundaries of the other moving and deforming phases present
$\mathbf{T}$ = stress tensor
$\overset{i}{\overline{\mathbf{v}}}$ = average over V of the velocity of phase i
V = volume of S
V_i = volume of pores containing phase i that are enclosed by S

Greek Letters

ρ = density; ρ_i is the density of phase i
φ = potential energy per unit mass
ψ = porosity. If there are n fluid phases present, then

$$\psi \equiv \frac{1}{V} \sum_{j=1}^{n} V_j$$

Special Symbols

$\overline{\ldots}$ average over V of a quantity associated with phase i. For example, $\overset{i}{\overline{B}}$ is an average over V of a quantity B associated with phase i:

$$\overset{i}{\overline{B}} \equiv \frac{1}{V} \int_{V_i} B dV$$

$<\ldots>^i$ Average over V_i of a quantity associated with phase i. For example, $<B>^i$ is an average over V_i of a quantity associated with phase i:

$$<B>^i \equiv \frac{1}{V_i} \int_{V_i} B dV$$

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Predicting Diffusion Rates in Some Materials with Abnormal Pore Structures

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Many models and methods have been proposed for a priori prediction of diffusion rates in porous materials. None of these is completely ideal, and it is important to define both their areas of suitability and their limitations. The most certain way of accomplishing this is to apply the models and methods to a large amount of experimental data, with the data covering as wide a range of process conditions, diffusing substances, and types of porous material as possible. This note applies the method currently most popular to recent data taken on some unusual porous materials over a reasonably wide range of temperature and pressure.

PREVIOUS WORK

Recently, the more popular models have been examined by Satterfield and Cadle (1968a, b), by Brown et al. (1969), and by Haynes and Brown (1971). An exhaustive review of various models and their applications has been presented by Haynes (1969). These studies concluded that the method proposed by Johnson and Stewart (1965) is currently the best available. The observed diffusion rates were usually within a factor of two of those predicted, and good extrapolation properties with pressure were obtained without exception. The latter property means that if one experiment (probably near atmospheric pressure) is used to obtain an empirical tortuosity factor, this tortuosity factor is reasonably valid at any other pressure.

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